A strong mathematics program at the middle level depends on a curriculum and textbooks that are aligned with NCTM’s standards, appropriate grouping of students, and solid instructional leadership.

Are middle school students learning the mathematics that will prepare them well for high school and a range of educational opportunities and career paths? Yes and no. Certainly some students are studying and learning important foundational mathematics. These students are intellectually engaged, see mathematics as important, work hard to succeed, and are generally flourishing within the middle school mathematics classroom learning environment. On the other hand, some students are not engaged, don’t view mathematics as important, and are not committing themselves to the effort needed to succeed. Many factors contribute to this dichotomy, including access to good teaching, support at home, and a rigorous and accessible mathematics curriculum.

The mathematics curriculum, although just one such factor, is the focus of this article because it is something every principal can influence. Ensuring that every middle level student has access to a coherent, rigorous, and focused
mathematics curriculum takes strong school leadership. In this article, we present the elements of such a curriculum and argue for careful selection and supported implementation of strong mathematics curriculum materials (i.e., textbooks) to assist teachers in raising expectations for all their students.

Revising the Curricula
The United States does not have a national curriculum as most industrialized and highly productive countries around the world do. Instead, control of what is taught in middle school mathematics classrooms resides at the state and local levels (Schmidt, Houang, & Cogan, 2002). This system, which has the advantage of empowering local constituents (e.g., administrators, board members, teachers, parents, and local community leaders), can and does in many cases lead to a very unfocused and generally unarticulated curriculum. Teachers teach what they have always taught or what they like the best or what the textbook says. Parents make demands on the basis of anecdotal instances (e.g., a mistake...
Suggesting that middle schools have either an algebra course or integrated strand of algebra is presenting a false dichotomy. In fact, either approach is possible. However, the goal should be that every student at the middle level have opportunities to learn the algebra expectations outlined in NCTM’s standards and that these opportunities not be delayed until they enroll in a course called algebra. Students in grades 6, 7, and 8 should be reasoning algebraically and beginning to use algebraic notation. Likewise, students throughout the middle level should explore rich, engaging real-world examples of algebra that will help them integrate algebraic concepts and skills with other branches of mathematics. Therefore, each mathematics course offered at the middle level should have a prominent strand of algebra, beginning in grade 6 and continuing through grade 8 so that by the end of middle school all students have learned the core algebra strand. To deliver this curriculum, careful attention to the selection of textbooks is necessary. There are different ways to organize mathematics courses to ensure that all students learn NCTM’s standards for algebra. It is important that middle level teachers have these learning expectations in mind as they make decisions about course organization and textbooks. If courses are organized and taught with these learning goals as the basis, then middle level mathematics programs will have made significant progress in preparing students for high school.

References
What Middle Level Students Need to Know

**Number and Operations Standard (Grades 6–8)**

Understand numbers, ways of representing numbers, relationships among numbers, and number systems:
- work flexibly with fractions, decimals, and percents to solve problems;
- compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line;
- develop meaning for percents greater than 100 and less than 1;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation;
- use factors, multiples, prime factorization, and relatively prime numbers to solve problems;
- develop meaning for integers and represent and compare quantities with them.

Understand meanings of operations and how they relate to one another:
- understand the meaning and effects of arithmetic operations with fractions, decimals and integers;
- use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals;
- understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.

Compute fluently and make reasonable estimates:
- select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods;
- develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use;
- develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

**Algebra Standard (Grades 6–8)**

Understand patterns, relations, and functions:
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- relate and compare different forms of representation for a relationship;
- identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

Represent and analyze mathematical situations and structures using algebraic symbols:
- develop an initial conceptual understanding of different uses of variables;
- explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope;
- use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;
- recognize and generate equivalent forms for simple algebraic expressions and solve linear equations.

Use mathematical models to represent and understand quantitative relationships:
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.

Analyze change in various contexts:
- use graphs to analyze the nature of changes in quantities in linear relationships.

Excerpt from *Principles and Standards for School Mathematics*, copyright 2000, National Council of Teachers of Mathematics.
Coherent curriculum. Rather than studying mathematical topics in isolation, a good mathematics curriculum links topics to other related mathematical ideas. The traditional mathematics curriculum (and many textbooks) organizes mathematics instruction into simple, one-day, bite-sized pieces. Every day the teacher presents a new topic and students practice working problems that are specific to that topic. Often, what they study is not connected to what they studied the previous day. Algebra and geometry are treated as separate fields of mathematics when, in fact, there is much overlap between the subjects and many ideas support one another. A coherent curriculum recognizes that students learn mathematics best when they see its relevance, are constantly relating new ideas to prior knowledge, and are able to engage in explorations that constantly elevate the sophistication of ideas. For many teachers, the ability to present a coherent curriculum depends on their knowledge of mathematics and a good set of curriculum materials that provide both a guide to sequencing ideas and specific activities that foster student thinking and link related ideas.

Engaging curriculum. What good is a well-articulated set of learning expectations if students aren’t engaged in studying the ideas? Mathematics is not a dead subject—it is used everyday to solve problems. Students need to see real uses of mathematics to understand its power and importance. There are many ways to engage students intellectually in the study of mathematics. Real problem contexts offer students practice working problems that are specific to that topic. Every day the teacher presents a new topic and students practice working problems that are specific to that topic. Often, what they study is not connected to what they studied the previous day. Algebra and geometry are treated as separate fields of mathematics when, in fact, there is much overlap between the subjects and many ideas support one another. A coherent curriculum recognizes that students learn mathematics best when they see its relevance, are constantly relating new ideas to prior knowledge, and are able to engage in explorations that constantly elevate the sophistication of ideas. For many teachers, the ability to present a coherent curriculum depends on their knowledge of mathematics and a good set of curriculum materials that provide both a guide to sequencing ideas and specific activities that foster student thinking and link related ideas.

Do Textbooks Matter?

Yes, textbooks matter! Textbooks are a much-used tool in mathematics classrooms. They help teachers organize and deliver instruction; serve as a source of problems for students to engage in and apply their knowledge; provide assessments to monitor student learning; and communicate to parents, often through homework assignments, what their children are studying. Educators place a great deal of trust in textbooks, so it is important that teachers and administrators regularly examine the content focus of district-adopted textbooks and the instructional strategies implicit within textbook lessons. Is the content and instructional focus consistent with goals for student learning? Does the textbook help teachers engage students in learning important mathematics or does it serve as a road map for passing through a host of isolated topics and procedures that will need to be revisited, yet again, next year?

Until recently, almost every middle level mathematics textbook looked alike—each lesson presented in a 2-page spread. The left-hand page presented problems with their solutions worked out. The right-hand page included lots of practice problems similar to the ones on the opposite page. Teachers used these practice problems for seat- and homework. The teachers role in presenting the lesson was to review and collect the homework, work several problems from the first page of the lesson on the board, answer student questions, then assign problems from the second page for students to work individually or in small groups. Textbooks were generally “teacher-proof”—anyone who could read could teach the lesson. However, the kind of instruction outlined in most traditional mathematics textbooks has not helped students learn or be motivated to study mathematics. Teaching in the way outlined by these textbooks did not lead to student learning, although it did provide the illusion of covering curriculum.

Today, new options in mathematics textbooks are available. Called “standards-based” textbooks, these new options were developed with support from the National Science Foundation to incorporate recommendations of the NCTM (Reys, Robinson, Sconiers, & Mark, 1999). The materials focus on learning expectations outlined in Figure 1. Interesting problems, often with real-world contexts, serve as the basis of instruction. Standards-based textbooks provide material (ideas, activities, problems) that teachers need to engage students in the study of mathematics. However, they...
are very different from traditional mathematics textbooks and teachers will need opportunities to carefully study them and work regularly with colleagues to learn new teaching strategies (see http://showmecenter.missouri.edu for more information about standards-based middle level mathematics curricula).

**Tracking**

To track or not to track—there is no one right solution for every situation. Yes, some students do benefit from hearing students at higher cognitive levels discuss their mathematical reasoning. Likewise, some students benefit from instruction that focuses on their specific needs. Although there is limited research evidence to guide the way in this matter, there is the wisdom of experience and common sense. Our own feeling is that it does not help low-ability students to isolate them from higher performing students. In fact, this strategy is likely counter-productive. Some range of ability levels within a class helps promote the emergence of various viewpoints and strategies within a class discussion. On the other hand, a group of highly performing mathematics students are likely to be ready to move faster and delve more deeply into topics beyond the expectations noted in Figure 1. For this small group of students, we recommend opportunities for acceleration and enrichment. For the larger group, we recommend a mixed-ability grouping organization. At the school level, this means that in some grades two course options are available to students, depending on their level of understanding and skill.

Many middle level teachers and administrators are feeling pressure to add an algebra course to the middle level curriculum. In fact, a significant amount of attention should be paid to the algebra strand in middle school. However, treating algebra as a strand in grades 6, 7, and 8 is an alternative to designating a course on algebra, particularly when this course is not appropriate or available to all students.

**Instructional Leadership**

Principals serve as key instructional leaders within a school. Their role includes hiring competent teachers and supporting their efforts for continual improvement. It also means calling for reform when needed. If students are not performing at high levels, then a comprehensive and coordinated effort is needed to examine the situation and plan for change. This effort should include the following elements:

- **Examine student performance.** Engage mathematics teachers and school leaders in reflecting on student performance. What are the strong and weak areas as noted by assessments and teacher experience? What strategies can be used to address the weak areas of performance?

- **Examine what is being taught.** What mathematics is being taught? How does it align with district, state or NCTM standards? Do all students have opportunities to learn this mathematics? Are there other ways to organize students to better deliver appropriate instruction?

**Examine the materials used to guide instruction, including textbooks.** Do district-adopted textbooks align with learning expectations? Do they provide ideas and resources to help teachers actively engage students?

**Provide opportunities for teachers to talk and learn from one another.** Do mathematics teachers have sufficient opportunities to talk with one another? To plan? To observe each other?

**Identify local expertise and leadership.** Is there good leadership within the group of mathematics teachers to facilitate improvements in teaching? What is needed to build or support this leadership? What outside expertise is needed to help guide and support mathematics faculty members?

**Serve as liaison between teachers and parents.** Be proactive in communicating with parents the goals and work of the mathematics faculty. Enlist their input. Help them understand their role in helping their children learn mathematics.

**Monitor improvement.** Develop a plan to help school leaders and teachers learn from what they are doing. It takes a strong team to provide students good learning experiences in mathematics in the middle grades. This team includes the principal, teachers, and parents. Each has a unique role in the process. The principal needs to provide the leadership to: establish clear learning goals in mathematics; select good textbook materials; and provide opportunities for teachers to continue to learn and grow professionally. Teachers must commit themselves to supporting all students in learning mathematics and be open to change if current techniques and resources are not adequate for this challenge. Parents need to support the school and regularly monitor their children’s efforts. Together, such a team can develop and implement a plan for school mathematics improvement. **PL**

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