**Lesson 13.6**  
**Rotations and Symmetry**

**BEFORE**  
You translated and reflected figures.

**Now**  
You’ll rotate figures and identify rotational symmetry.

**WHY?**  
So you can describe how a CD tray rotates, as in Ex. 17.

**Family Crests**  
A family crest is a design that symbolizes a family’s heritage. An example of a family crest for a Japanese family is shown. In Example 4, you will look at the rotational symmetry of the design.

A **rotation** is a transformation in which a figure is turned about a fixed point, called the **center of rotation**. The **angle of rotation** is formed by rays drawn from the center of rotation through corresponding points on an original figure and its image. The direction of rotation can be clockwise or counterclockwise. In a rotation, a figure and its image are congruent.

**Example 1**  
**Identifying Rotations**

Tell whether the transformation is a rotation about the origin. If so, give the angle and direction of rotation.

**Solution**

a. 90° clockwise rotation  
   
b. Not a rotation  
   
c. 180° rotation in either direction
90° Rotations  In this book, all rotations in the coordinate plane are centered at the origin. You can use coordinate notation to describe a 90° rotation of a figure about the origin.

**90° clockwise rotation**

Switch the coordinates, then multiply the new \( y \)-coordinate by \(-1\).

\[(x, y) \rightarrow (y, -x)\]

**90° counterclockwise rotation**

Switch the coordinates, then multiply the new \( x \)-coordinate by \(-1\).

\[(x, y) \rightarrow (-y, x)\]

### Example 2  Rotating a Triangle

Draw \(\triangle ABC\) with vertices \(A(-3, 4), B(-2, 3),\) and \(C(-2, 1)\). Then find the coordinates of the vertices of the image after a 90° clockwise rotation, and draw the image.

**Solution**

First draw \(\triangle ABC\). Then, to rotate \(\triangle ABC\) 90° clockwise, switch the coordinates and multiply the new \( y \)-coordinate by \(-1\).

<table>
<thead>
<tr>
<th>Original</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x, y))</td>
<td>((y, -x))</td>
</tr>
<tr>
<td>(A(-3, 4))</td>
<td>(A'(4, 3))</td>
</tr>
<tr>
<td>(B(-2, 3))</td>
<td>(B'(3, 2))</td>
</tr>
<tr>
<td>(C(-2, 1))</td>
<td>(C'(1, 2))</td>
</tr>
</tbody>
</table>

Finally, draw \(\triangle A'B'C'\), as shown.

**Checkpoint**

1. In Example 2, find the coordinates of the image of \(\triangle A'B'C'\) after a 90° clockwise rotation, and draw the image \(\triangle A''B''C''\). How are the coordinates of the vertices of \(\triangle A''B''C''\) related to those of \(\triangle ABC\)?

2. In Example 2, find the coordinates of the image of \(\triangle ABC\) after a 90° counterclockwise rotation, and draw the image.

3. **Critical Thinking**  A figure lies in the third quadrant of a coordinate plane. In what quadrant does the image lie after a 90° clockwise rotation? after a 90° counterclockwise rotation?
180° Rotations  To rotate a point 180° about the origin, multiply each coordinate by $-1$. The image is the same whether you rotate the figure clockwise or counterclockwise. $(x, y) \rightarrow (-x, -y)$

Example 3  Rotating a Triangle

Draw $\triangle MNP$ with vertices $M(1, -2)$, $N(4, -1)$, and $P(2, -3)$. Then find the coordinates of the vertices of the image after a 180° rotation, and draw the image.

Solution

First draw $\triangle MNP$. Then, to rotate $\triangle MNP$ 180°, multiply the coordinates by $-1$.

<table>
<thead>
<tr>
<th>Original</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x, y)$</td>
<td>$(-x, -y)$</td>
</tr>
<tr>
<td>$M(1, -2)$</td>
<td>$M'(-1, 2)$</td>
</tr>
<tr>
<td>$N(4, -1)$</td>
<td>$N'(-4, 1)$</td>
</tr>
<tr>
<td>$P(2, -3)$</td>
<td>$P'(-2, 3)$</td>
</tr>
</tbody>
</table>

Finally, draw $\triangle M'N'P'$, as shown.

Checkpoint

4. Draw $\triangle DEF$ with vertices $D(-6, -1)$, $E(0, -2)$, and $F(-5, -4)$. Then find the coordinates of the vertices of the image after a 180° rotation, and draw the image.

Rotational Symmetry  A figure has rotational symmetry if a rotation of 180° or less clockwise (or counterclockwise) about its center produces an image that fits exactly on the original figure.

Example 4  Identifying Rotational Symmetry

The family crest shown on page 741 has rotational symmetry for a 90° or 180° clockwise (or counterclockwise) rotation.

In the Real World

Family Crests  The family crest shown above has rotational symmetry for a 120° clockwise rotation. Give another angle and direction of rotation that produce rotational symmetry for the crest.
Chapter 13  Angle Relationships and Transformations

Guided Practice

Vocabulary Check
1. How are rotational symmetry and line symmetry different?
2. Use coordinate notation to describe a 90° counterclockwise rotation.

Skill Check
Tell whether the transformation is a rotation about the origin. If so, give the angle and direction of rotation.

3. 4. 5.

6. Draw \(\triangle ABC\) with vertices \(A(3, 2), B(5, 1),\) and \(C(6, 4)\). Then find the coordinates of the vertices of the image after a 90° clockwise rotation, and draw the image.

7. Error Analysis Describe and correct the error in finding the coordinates of the vertices of the image of \(\triangle ABC\) after a 90° clockwise rotation.

Practice and Problem Solving

Tell whether the transformation is a rotation about the origin. If so, give the angle and direction of rotation.

8. 9. 10.

The vertices of a polygon are given. Draw the polygon. Then find the coordinates of the vertices of the image under the specified rotation, and draw the image.

11. \(A(1, 3), B(5, 6), C(5, 3)\); 90° counterclockwise rotation
12. \(P(-6, 2), Q(-3, 4), R(-1, 3), S(-5, 0)\); 90° clockwise rotation
13. \(J(2, -1), K(4, -1), L(4, -5), M(3, -6), N(2, -5)\); 180° rotation
Tell whether the figure has rotational symmetry. If so, give each angle and direction of rotation that produce rotational symmetry.

14. 15. 16.

17. **CD Player** Your CD player can hold five compact discs on a rotating tray like the one shown.

   a. Does the tray have rotational symmetry? Explain.
   b. The tray can move only clockwise. A CD in position 1 is currently playing. How many degrees must the tray rotate to play a CD in position 3?

18. Draw \(\triangle JKL\) with vertices \(J(-6, -5), K(-4, -3),\) and \(L(-2, -3)\).

   a. You rotate \(\triangle JKL\) 90° clockwise, then you rotate its image 180°. Find the coordinates of the final image. Then draw the image.
   b. **Critical Thinking** Use coordinate notation to describe how to rotate \(\triangle JKL\) to the final image in part (a) using one rotation.

19. **Extended Problem Solving** The table shows the first four regular polygons that have an even number of sides.

<table>
<thead>
<tr>
<th>Regular polygon</th>
<th>Quadrilateral</th>
<th>Hexagon</th>
<th>Octagon</th>
<th>10-gon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sides</td>
<td>4</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Angles of rotation (in either direction)</td>
<td>90°, 180°</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

   a. Copy and complete the table by finding the number of sides of each regular polygon and the angles of rotation that produce rotational symmetry.
   b. **Compare** How is the number of sides related to the number of angles of rotation?
   c. **Predict** Add a column in the table for a regular 16-gon.

**Draw the polygon shown. Then find the coordinates of the vertices of the final image after the specified transformations, and draw the final image.**

20. Rotate the polygon 180°, then reflect the image in the \(y\)-axis.

21. Rotate the polygon 90° clockwise, then reflect the image in the \(x\)-axis.

22. Rotate the polygon 90° counterclockwise, then translate the image using \((x, y) \rightarrow (x + 3, y + 4)\).
23. **Tessellations** You can rotate and translate a quadrilateral to create a tessellation, as shown. Use this approach to create a tessellation using a different quadrilateral.

1. Draw any quadrilateral and rotate it 180° about the midpoint of one of its sides.

2. Translate the new figure repeatedly to form a tessellation.

24. **Challenge** A triangle is rotated 90° clockwise about the origin, then its image is translated using \((x, y) \rightarrow (x + 3, y - 1)\). The coordinates of the vertices of the final image are \((1, -4), (3, -2),\) and \((6, -5)\). Find the coordinates of the vertices of the original triangle.

## Mixed Review

A map has a scale of 1 inch : 50 miles. Use the given map distance to find the actual distance. *(Lesson 6.6)*

25. 1.5 inches  
26. 3 inches  
27. 6 inches  
28. 8.5 inches  

29. Draw \(\triangle PQR\) with vertices \(P(-5, -4), Q(-3, 0),\) and \(R(-1, -3)\). Then find the coordinates of the vertices of the image after a reflection in the \(y\)-axis, and draw the image. *(Lesson 13.5)*

## Standardized Test Practice

30. **Extended Response** \(\triangle ABC\) has vertices \(A(-6, 2), B(-2, 5),\) and \(C(-4, 1)\).

   a. Find the coordinates of the vertices of the image of \(\triangle ABC\) after a 90° clockwise rotation about the origin, and draw the image \(\triangle A'B'C'\). Then find the coordinates of the vertices of the image of \(\triangle A'B'C'\) after a reflection in the \(x\)-axis, and draw the image \(\triangle A''B''C''\).

   b. If you switch the order of the transformations, is the image \(\triangle A''B''C''\) the same? Justify your answer.

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**Treasure Hunt**

You are located at the point \((3, 4)\) in a coordinate plane. You need to find your way to a treasure chest. Starting at \((3, 4)\), move from one image point to the next by following the order of the transformations listed. The final image point is the location of the treasure chest.

1. Rotate 180°.
2. Reflect in the \(y\)-axis.
3. Translate 5 units to the left and 4 units up.
4. Reflect in the \(x\)-axis.
5. Rotate 90° clockwise.